1 Design of Single Sampling Plans for Defectives

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1.1 Abstract

A procedure to design a single sampling plan for defectives for the purpose of distinguishing good lots from bad lots is presented. The procedure requires input data on what fractions defective correspond to good and bad lots and the desired probability of accepting such lots. The procedure outputs the sample size and acceptance number that meet or exceed these requirements. Examples are provided.

1.2 Key Words

acceptance sampling, sampling plan, single sampling plan, defective, nonconforming unit, fraction defective, MIL-STD-105, ANSI/ASQ Z1.4, Larson nomogram, binomial distribution

1.3 Introduction

Acceptance sampling is a common quality management technique used to distinguish good lots of material from bad lots. A bad lot is a lot that contains too many defective parts. The technique relies on the use of statistically determined sample sizes and acceptance criteria to safely determine which lots are good and which are bad. Acceptance sampling does not control product quality, it only provides a method of sentencing lots, i.e. calling them good or bad.

This article outlines a procedure to design one of the simplest and most commonly used acceptance sampling plans, a single sampling plan (SSP) to screen for lots with an excessive number of defective parts. In SSPs for defectives a sample is drawn from a lot and inspected. If the number of defective parts in the sample is sufficiently small the entire lot is accepted. If too many defectives are found in the sample the entire lot is rejected.

1.4 Where the Technique is Used

This technique is used when a single lot or a series of lots is submitted for lot acceptance and a single sampling plan which limits Type 1 (rejecting good stuff) and Type 2 (accepting bad stuff) errors is desired. The technique is commonly used by the manufacturer of a product before product is released to the customer and by a customer before received product is accepted for use.

1.5 Data

The design of the SSP requires the following information from management:

- The allowable fraction defective in a good lot and the corresponding probability of accepting such lots.
- The fraction defective that makes a lot bad and the corresponding probability of accepting such lots.

1.6 Assumptions

- The lot consists of many parts of the same type.
- Parts can be classified as defective or not defective.
- Parts are independent of each other, that is, whether a part is good or bad does not affect the previous or following part when it is made or inspected.
- Every part has the same probability of being defective.
- Inspection to classify parts as defective or not defective is 100% accurate.
- Operators are not biased toward accepting or rejecting parts.
- Parts for the sample are drawn randomly so that the sample is representative of the lot.

1.7 Hypotheses Tested

The single sampling plan corresponds to testing the following hypotheses:

$$\begin{aligned} H_0 &: \text{ The lot is good (i.e. contains very few defective parts)} \\ H_A &: \text{ The lot is bad (i.e. contains too many defective parts)} \end{aligned}$$

In a rigorous statistical sense these hypotheses may be expressed as:

$$\begin{aligned} H_0 &: p \leq p_\alpha \\ H_A &: p > p_\alpha \end{aligned}$$

where p_α is the allowable fraction defective in a good lot.

1.8 Analysis

1.8.1 Introduction to the Larson Nomogram

This technique requires the use of the Larson nomogram. Nomograms are considered to be an old technology and have been largely replaced by software. Still, there are many problems, such as this one, which are simply and elegantly solved with nomograms. Software is available to design single sampling plans to control lot fraction defective but the use of the Larson nomogram is equivalent, instructive, and instills confidence in the SSP designer.

The Larson nomogram is a graphical method of determining approximate cumulative binomial probabilities for given n and p binomial parameters:

$$b\left(c; n, p\right) = \sum_{x=0}^{c} \binom{n}{x} p^{x} \left(1-p\right)^{n-x}$$

where n is the sample size, p is the fraction defective, x is the number of successes, c is the cumulative number of successes, and $b\left(c; n, p\right)$ is the cumulative binomial probability. The Larson nomogram is shown in Figures 1 and 2 for $0.001 \leq p \leq 0.05$ and $0.01 \leq p \leq 0.50$, respectively. The four quantities c, n, p, and $b\left(c; n, p\right)$ are represented on the nomogram. The n and c values are determined in the body of the nomogram. The left hand vertical axis of the nomogram is p and the right hand vertical axis is $b\left(c; n, p\right)$. A cumulative binomial probability is found by finding the point in the body of the nomogram that corresponds to the desired n and c and then extending a line through that point from the desired p on the left hand vertical axis. The required $b\left(c; n, p\right)$ is found where the extension of the line crosses the right hand vertical axis.

Example: *Use the Larson nomogram to find* $b\left(c = 3; n = 100, p = 0.05\right)$.
Solution: *Figure 3 shows how to use the Larson nomogram to solve the example problem. The circled point in the body of the nomogram identifies the point* $(c = 3, n = 100)$. *The value* $p = 0.05$ *is circled on the left hand vertical axis and a line is drawn between the two points. Where the line crosses the right hand vertical axis we find* $b\left(3; 100, 0.05\right) = 0.26$.

1.8.2 Procedure for Designing the SSP

1. Determine what fraction of a lot can be defective and still have the lot considered to be good. Call this fraction defective p_α. p_α should be small.

2. Decide what probability of accepting (P_a) good lots with fraction defective p_α is desired. Call this probability $P_a = 1 - \alpha$. α corresponds to the probability of rejecting good stuff (a Type 1 error) and should be small. α is also called the manufacturer's risk.

3. Determine what fraction of a lot must be defective so that the lot is considered to be bad. Call this fraction defective p_β. p_β should be small, but larger than p_α.

4. Decide what probability of accepting bad lots with fraction defective p_β can be tolerated. Call this probability $P_a = \beta$. β corresponds the probability of accepting bad stuff (a Type 2 error) and should be small. β is also called the consumer's risk.

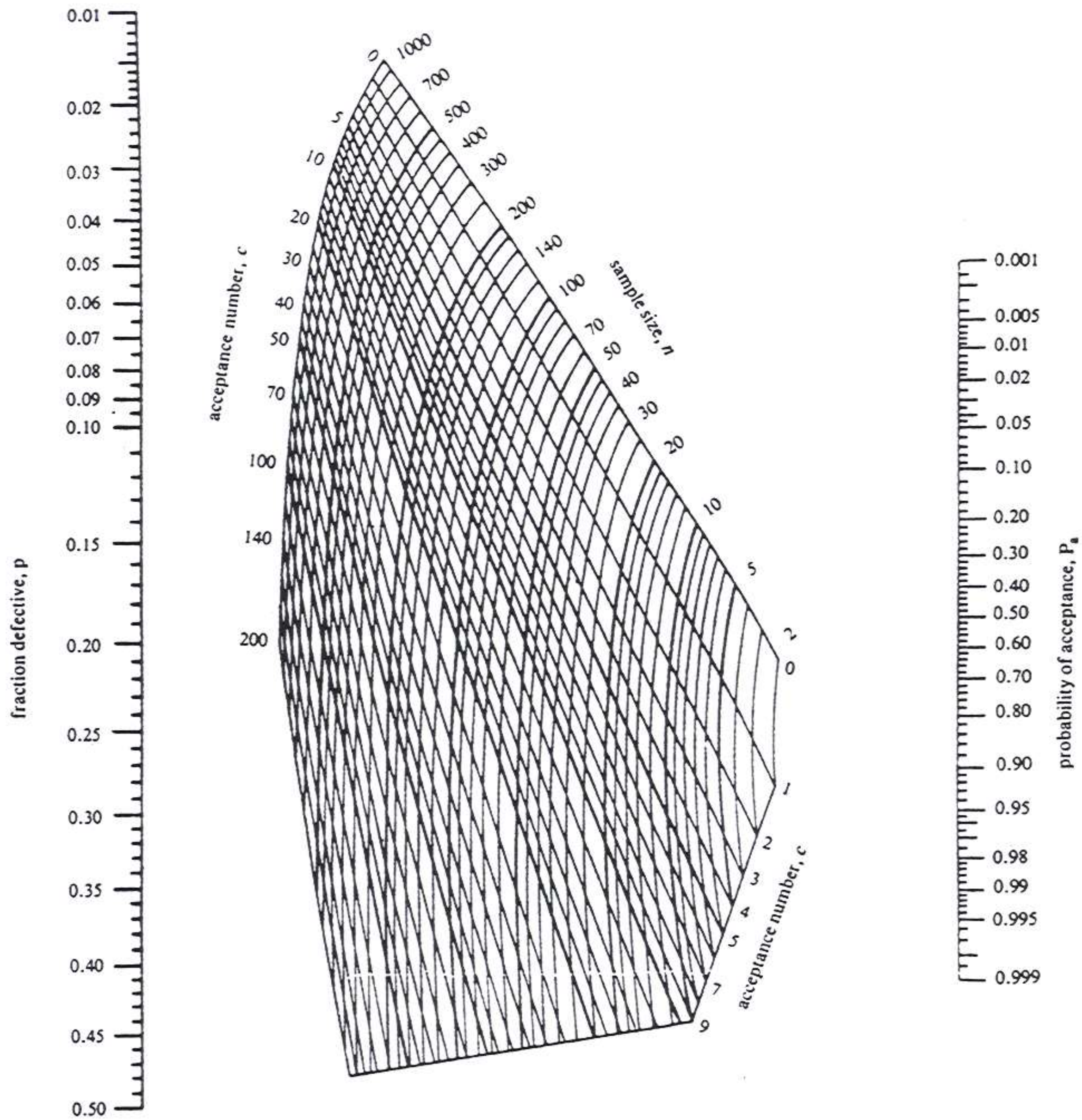

Figure 1: Larson's nomogram for $b\,(c; n, p)$ where $0.01 \leq p \leq 0.5$.

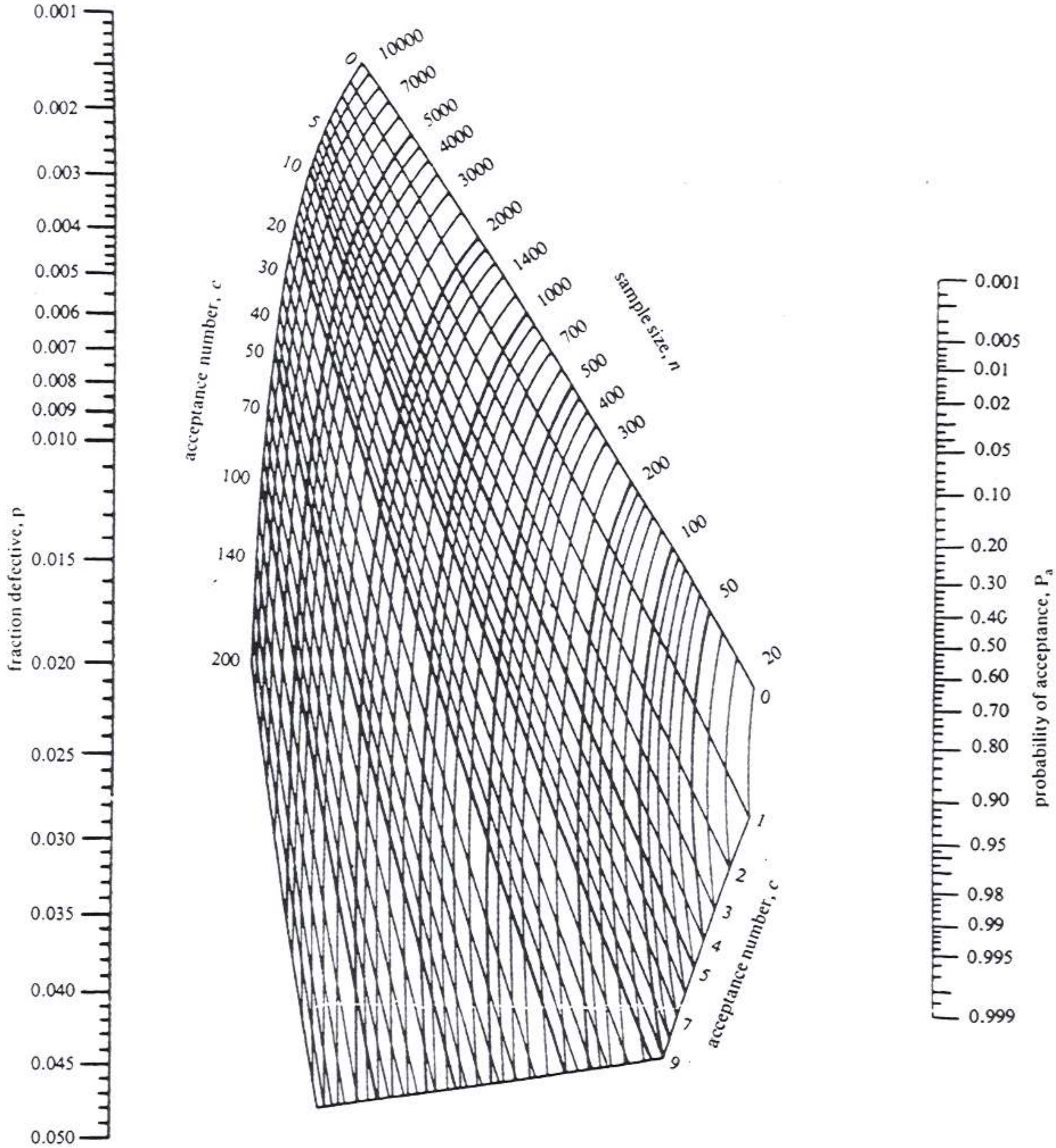

Figure 2: Larson's nomogram for $b\,(c; n, p)$ where $0.001 \leq p \leq 0.05$.

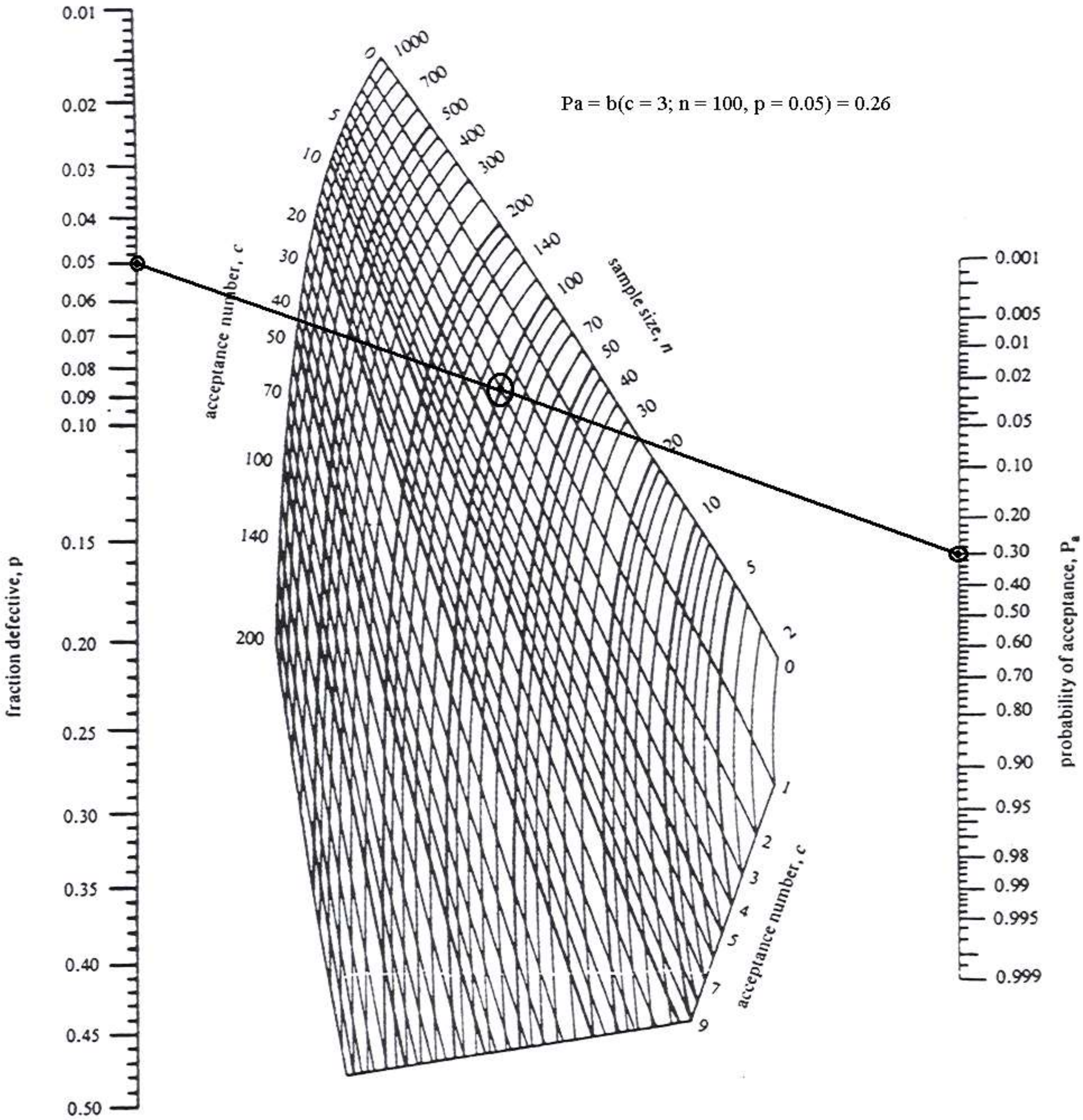

Figure 3: Larson's nomogram solution for $b\,(c = 3; n = 100, p = 0.05)$.

5. Circle the points corresponding to p_α and $P_a = 1 - \alpha = b(c; n, p)$ on the Larson nomogram and connect them with a straight line.

6. Circle the points corresponding to p_β and $P_a = \beta = b(c; n, p)$ on the Larson nomogram and connect them with a straight line.

7. Read the SSP parameters n and c from the body of the nomogram. The desired SSP uses a sample of size n and the maximum allowable number of defective parts in the sample to accept the lot is c.

8. Operate the SSP by drawing a random sample of size n from the lot. Inspect the lot and if the number of defectives in the sample, x, is less than or equal to c (that is, $0 \leq x \leq c$) then accept the lot, otherwise ($c < x \leq n$) reject the lot.

Example: *Design the SSP that will accept* 95% *of the lots containing* 2% *defective parts and will accept only* 5% *of the lots containing* 9% *defective parts.*
Solution: *From the problem statement we have* $P_a = 1 - \alpha = 0.95$ *so* $\alpha = 0.05$ *and* $p_\alpha = 0.02$. *We also have* $P_a = \beta = 0.05$ *and* $p_\beta = 0.09$. *The lines corresponding to* $(p_\alpha = 0.02, P_a = 0.95)$ *and* $(p_\beta = 0.05, P_a = 0.09)$ *are drawn on the Larson nomogram in Figure 4. The lines intersect at* $n = 100$ *and* $c = 4$. *The sampling plan is operated by drawing random samples of size* $n = 100$ *from the lot and inspecting the sample. The lot is accepted if* $c = 4$ *or fewer defectives are found in the sample. Otherwise the lot is rejected.*

Example: *Construct the operating characteristic (OC) curve,* P_a *vs.* p, *for the SSP in the preceding example. Confirm that the points* $(p_\alpha = 0.02, P_a = 0.95)$ *and* $(p_\beta = 0.09, P_a = 0.05)$ *are on the OC curve.*
Solution: *Since the SSP is characterized by* $n = 100$ *and* $c = 4$ *we can find the* P_a *value for any* p *from the Larson nomogram. Several such points are determined in Figure 5 and the requested OC curve is shown in Figure 6. The points* $(p_\alpha = 0.02, P_a = 0.95)$ *and* $(p_\beta = 0.09, P_a = 0.05)$ *are circled and fall on the OC curve.*

1.9 Discussion

1.9.1 Mathematical Interpretation

By picking the two points $(p_\alpha, P_a = 1 - \alpha)$ and $(p_\beta, P_a = \beta)$ we are defining the shape of an OC curve. A unique choice of n and c meets these conditions. The intersection of the lines on the Larson nomogram corresponding to these conditions is the mathematical equivalent of simultaneously solving the two equations:

$$b(c; n, p_\alpha) = 1 - \alpha$$

and

$$b(c; n, p_\beta) = \beta$$

for c and n.

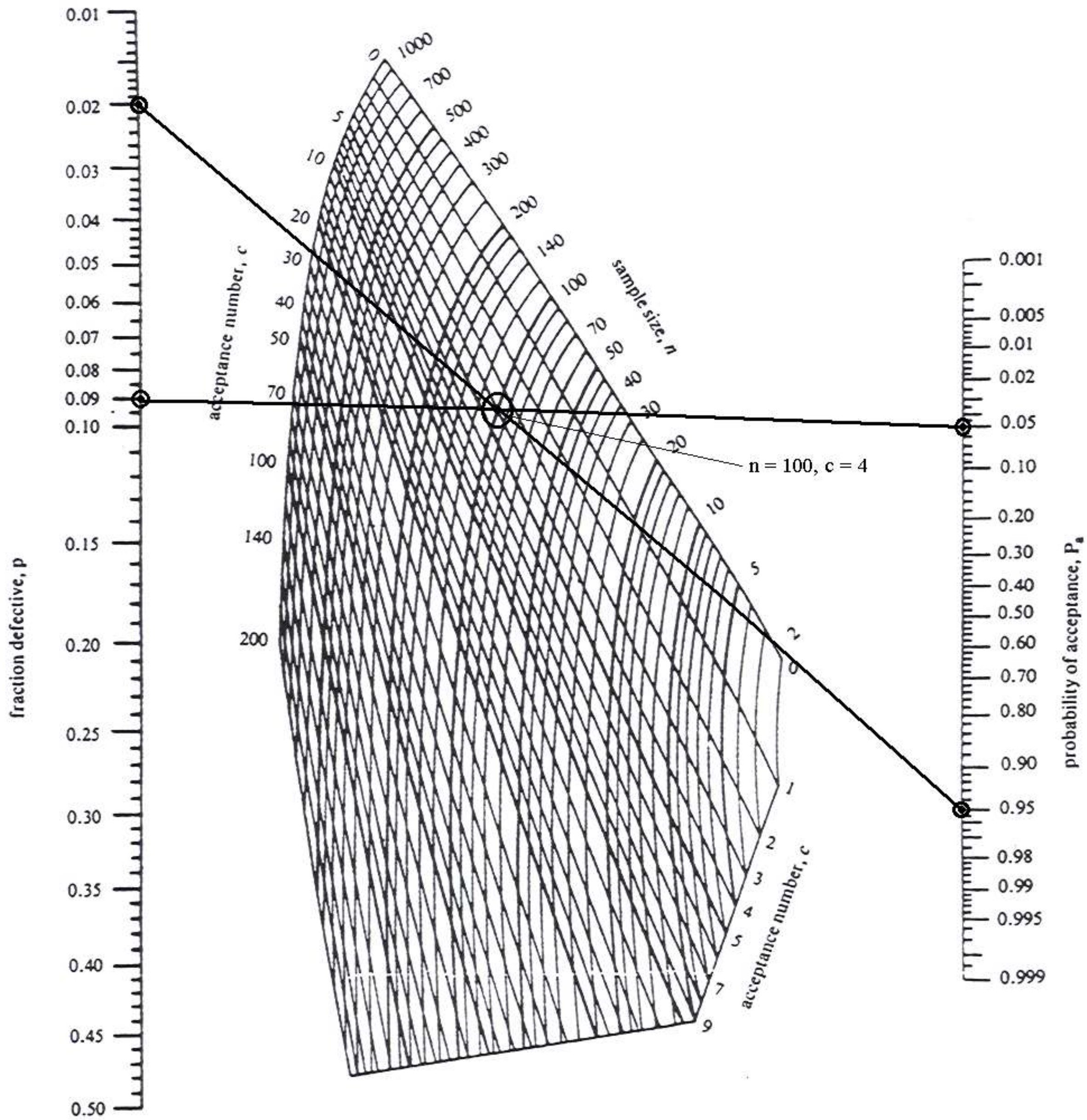

Figure 4: Sampling plan solution for $(p, P_A) = (0.02, 0.95)$ and $(p, P_A) = (0.09, 0.05)$.

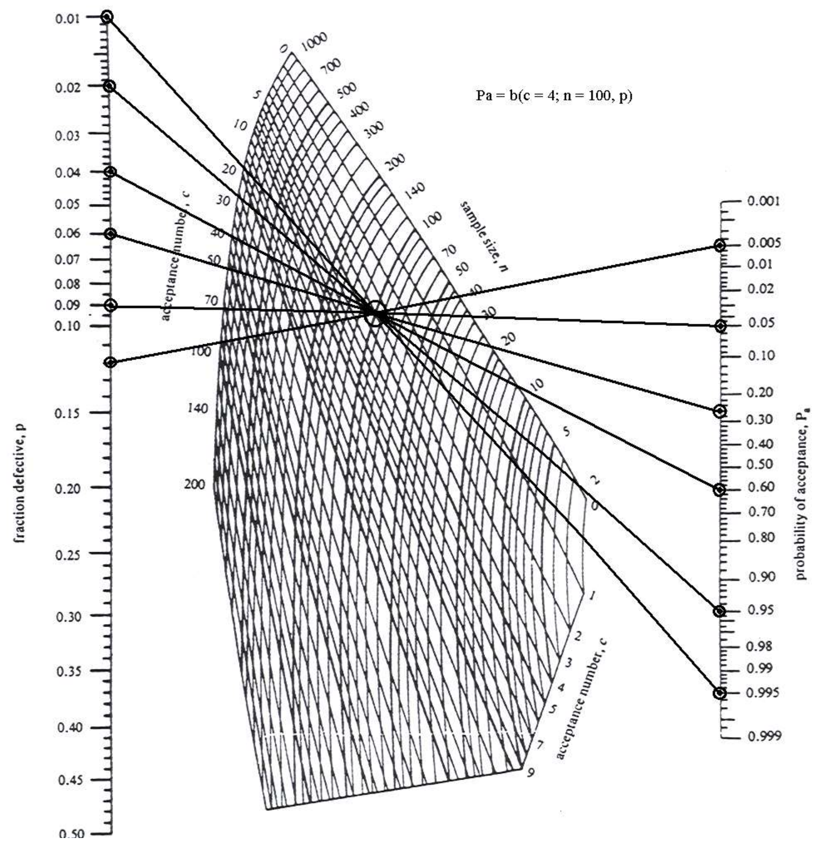

Figure 5: Obtaining points for an OC curve.

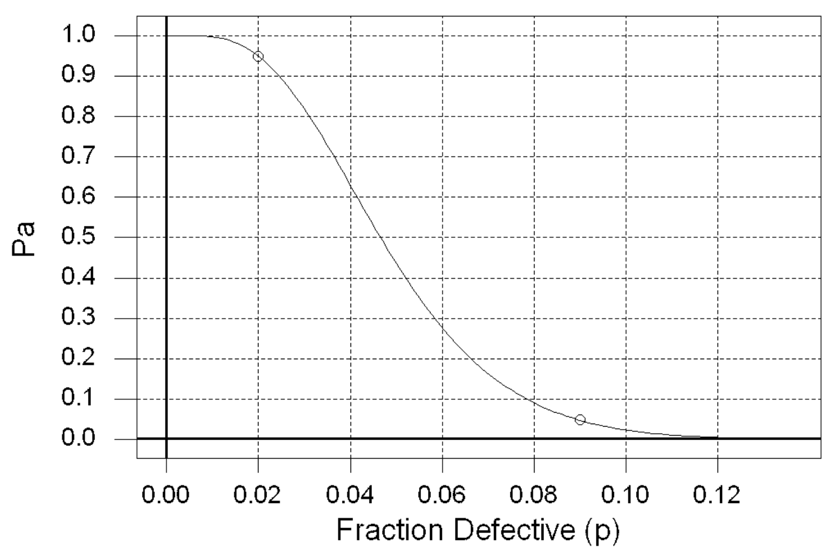

Figure 6: OC curve for $n = 100$, $c = 4$.

1.9.2 When Lines Don't Cross at an Integer Value of c

When the two lines drawn on the Larson nomogram intersect between contours of c it is necessary to move the point of intersection to one of the adjacent contours. This must be done carefully - even a small shift in the (n, c) operating point can cause a large change in the corresponding α and/or β. Decreases in α and/or β are acceptable but any increases must be avoided or, at the least, carefully controlled.

To obtain a safe SSP that meets or exceeds the original requirements hold one of the lines drawn on the nomogram fixed, then swing the right end of the other line outward so that the endpoints of the two lines on the P_a scale get farther apart. (The left end of the second line, the one being swung, stays fixed at its original p value.) Swing the line until the point of intersection reaches the nearest c contour. This procedure guarantees that α and β will be less than or equal to the specified values.

Example: *Design the SSP that meets or exceeds the following conditions:* $(p = 0.02, P_a = 0.98)$ *and* $(p = 0.20, P_a = 0.10)$*. If necessary, hold* α *fixed and improve* β*.*
Solution: *The two lines corresponding to the specifications in the problem statement are drawn on the Larson nomogram in Figure 7. The lines intersect between the* $c = 1$ *and* $c = 2$ *contours so it is necessary to deviate from the original spec. If the first line* $(0.02, 0.98)$ *is held fixed and we swing the right end of the second line upward we obtain* $n = 30$ *and* $c = 2$*. This design decreases the type 2 error rate from* $P_a = 0.10$ *to* $P_a = 0.04$*.*

1.9.3 Getting the Required Information from Management

It can be difficult to get the information necessary to design the SSP from management. Remember, they have a very limited skill set and you will have to ask very specific questions. Try these:

- How large can the lot fraction defective be and still consider the lot to be good? 1%? 2%? 5%? 0% is not an acceptable answer. (When you get them to answer this question you know p_α.)
- What probability of accepting lots with fraction defective p_α is required? Remember, these are good lots and the lots we don't accept are scrapped (or 100% inspected or shipped back to the supplier). 95%? 98%? 99%? 100% is not an acceptable answer. (The answer to this question is $P_a = 1 - \alpha$.)
- What fraction of a lot must be defective for the lot to be considered bad? 5%? 8%? 10%? (The answer to this question is p_β.)
- What probability of accepting lots with fraction defective p_β can be tolerated? Remember, these are bad lots and will cause problems. 5%? 10%? 15%? (The answer to this question is $P_a = \beta$.)

1.9.4 What If the Sample Size is Greater Than the Lot Size?

When the lot size is small the sample size may exceed it. In this case it is necessary to inspect all of the parts in the lot (100% inspection).

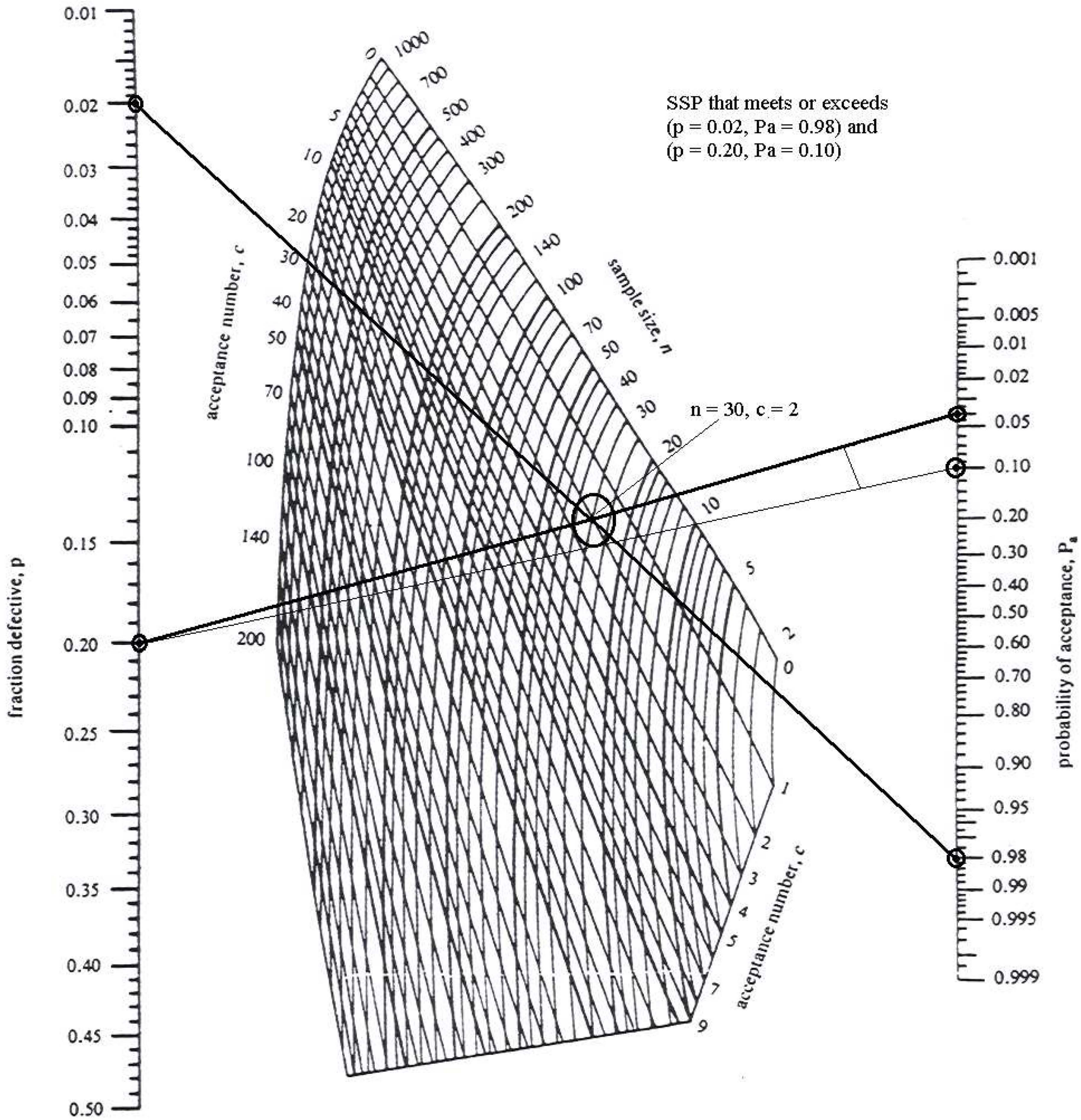

Figure 7: Adjusting the acceptance number c.

1.9.5 Other Sampling Plans for Defectives

More economical sampling plans than SSPs are available. These sampling plans are called double sampling plans (DSP), multiple sampling plans (MultSP), and sequential sampling plans (SeqSP). In general these plans use two or more small samples from a lot to decide whether the lot should be accepted or rejected. Very good or very bad lots are accepted or rejected after only the first or first few samples are drawn. This gives these plans the advantage of requiring that fewer units be inspected, on average, than the corresponding SSP with the same OC curve. Their disadvantage is that they are harder to design and administrate than SSPs. Special tables and design algorithms for DSPs, MultSPs, and SeqSPs can be found in the references.

1.9.6 Relationship to ANSI/ASQ Z1.4 (or MIL-STD-105)

ANSI/ASQ Z1.4 is an acceptance sampling system for defectives. It is comprised of individual sampling plans that are to be used together. Such a collection of sampling plans is called a sampling scheme.

The procedure outlined in this article gives sampling plans closely related to the plans cataloged in ANSI/ASQ Z1.4, however, Z1.4 uses an additional constraint that increases the sample size as the lot size increases. Z1.4 also provides the SSP, DSP, and MultSP for a given set of conditions.

1.10 References

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